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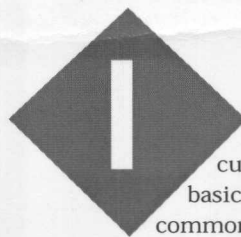
## Technically Speaking

James Antonakos

### Passive and Active Filters

Passing and Rejecting Signals Based on Their Frequency

Who knew that brewing a cup of coffee could be the first step in understanding a basic engineering concept? Well, James takes us through the different blends of filters, whetting our appetite for more. He also challenges us to taste them all-after all, variety is the spice of life.



Last month, I discussed some of the basic aspects of the common emitter amplifier. This included mention of a high-pass filter in the amplifier, formed by the input coupling capacitor and input resistance of the amplifier. This month, let's take a step farther back and review the theory behind high-pass, low-pass, and other types of filters.

#### WHAT AND WHY?

If you are familiar with the filters used for making coffee, then you may understand the principle behind the filter. In the case of a coffee filter, the large coffee grains are blocked, and the smaller molecules of water pass through. Such is the case with an electronic filter. Some signals pass through the filter and others are blocked based on their frequency.

There are many applications that require the use of one or more types of filters. Radio, television,

and other communications equipment make use of filters to capture and process specific audio and video signals. A speaker employs a filter network to control its woofer and tweeter. Other filters eliminate 60-cycle hum and other unwanted noise.

#### A FILTER BY ANY OTHER NAME

There are four basic filter types (see Table 1). I will concentrate mostly on low-pass and high-pass filters. With a little creative thinking, it's not difficult to see how band-pass and band-reject (or notch) filters are just combinations of low-pass and high-pass filters.

#### PASSIVE VERSUS ACTIVE

A passive filter is a filter that does not have any active components like transistors or op-amps. For example, a simple low-pass filter that is built out of a resistor and capacitor is a passive filter. The output signal can never have more power than the input signal, because resistors, capacitors, and coils have no gain themselves.

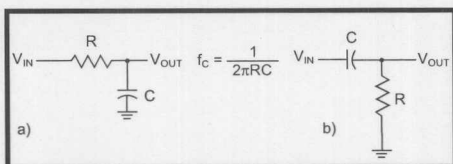
A filter that incorporates one or more transistors or operational amplifiers is an active filter. Here, the output signal can be larger than the input signal when it is passed by the filter.

#### WHAT'S THE FREQUENCY, KENNETH?

In the case of the low-pass and high-pass filters, you refer to the corner frequency of the filter as the frequency where the input signal begins getting passed or rejected. For the passive filters (see Figures 1a and b), the corner frequency depends on the value of resistor and capacitor used in

Filter	Operation
Low-pass	Passes low frequencies
High-pass	Passes high frequencies
Band-pass	Passes a range of frequencies
Band-reject	Rejects a range of frequencies

Table 1—The four basic types of filters can be seen here. Each filter reacts differently over its frequency response.



Figures 1a and b—Passive low-pass and high-pass filters' frequencies are determined by R and C.

the filter. For example, if  $R = 1$  kilohm and  $C = 0.1$   $\mu$ F, the corner frequency is 1591 Hz.

Let's think about this frequency, 1591 Hz. You can't hook a frequency counter up to the filter and measure the corner frequency. Instead, the corner frequency tells you something about the gain of the filter. In fact, the passive low-pass and high-pass filters both have a gain of 0.707 when the input signal has a frequency equal to the corner frequency (see Figures 2a and b).

The low-pass filter has a gain of 1.0 below its corner. This indicates that the output signal has the same amplitude as the input signal at frequencies below the corner. As frequency increases, the output begins to roll off, lowering to 0.707 of its maximum value at the corner and going even lower at frequencies above the corner.

The opposite is true for the high-pass filter. At frequencies below the corner, there is little output amplitude. Only when the input frequency is at or above the corner does the output become large (0.707 or more of its maximum).

## A CLOSER LOOK

Take, for example, a low-pass filter with a corner frequency of 1591 Hz and apply a 5- $V_{RMS}$ , 2-KHz input signal to it. What is the output amplitude?

In order to determine the answer, you must know the relationship between the gain and frequency in the filter. Figure 3 shows how the gain formula for the passive low-pass filter is derived.

Think of the passive low-pass filter in Figure 1 as a voltage divider. Some of the input voltage will drop across the resistor, and the rest will drop across the capacitor. Though the resistor value is fixed, the capacitive reactance will vary with frequency

(getting smaller as frequency increases). So, the action of the voltage divider will vary, as will the output amplitude.

Starting with the basic voltage divider formula in Figure 3, substitute the formula for capacitive reactance and then use algebra to simplify things. Notice that the resulting formula shows the gain as a function of the input frequency ( $f$ ) and the corner frequency ( $f_c$ ). The ratio of these two frequencies is what drives the gain and phase shift of the filter.

To check the formula, note that as  $f$  increases, the ratio of  $f$  to  $f_c$  gets larger. This makes the denominator of the gain fraction larger, which makes the overall fraction smaller. Thus, as  $f$  increases, the gain gets smaller, as it should in a low-pass filter.

Photo 1 shows a screen shot of Excel providing a suitable range of input frequencies to the low-pass filter and the associated output values. The gain in dB is easily found by multiplying the base-10 log of the filter gain by 20. For example,  $20 \times (\text{base-10 log of } 0.707) = -3$  dB.

Do you see that almost the full range of gain (from one down to zero) is contained in a frequency range that goes from a decade below the corner (159 Hz) to a decade above the corner (15.9 KHz)? This is important to remember.

Also, do you see that that gain has fallen by 20 dB in the second decade

	A	B	C	D
	Freq	Gain	Gain (dB)	Phase
1				
2				
3	100	0.998031	-0.01712	-3.59651
4	159	0.995043	-0.04316	-5.70703
5	200	0.992191	-0.06809	-7.1649
6	300	0.982689	-0.15173	-10.6784
7	400	0.969819	-0.26619	-14.1125
8	500	0.953999	-0.40905	-17.4453
9	600	0.935675	-0.5775	-20.6625
10	700	0.915324	-0.76851	-23.7483
11	800	0.893414	-0.97894	-26.6945
12	1000	0.846655	-1.44592	-32.1500
13	1591	0.707107	-3.0103	-45
14	2000	0.622545	-4.11658	-51.4978
15	3000	0.460524	-6.59537	-62.0615
16	4000	0.369588	-8.64565	-68.3088
17	5000	0.303219	-10.3649	-72.3489
18	7000	0.221633	-13.0873	-77.195
19	8000	0.195055	-14.1969	-78.7521
20	9000	0.174079	-15.1851	-79.975
21	10000	0.157124	-16.0752	-80.96
22	15910	0.099504	-20.0432	-84.2094
23				

Photo 1—Here you can see Excel performing gain and phase calculations on the low-pass filter.

of frequencies (1591 Hz to 15.9 KHz)? This is a characteristic of a first-order filter. A second-order filter has a roll-off of -40 dB/decade. A third-order filter would be -60 dB/decade, and so on.

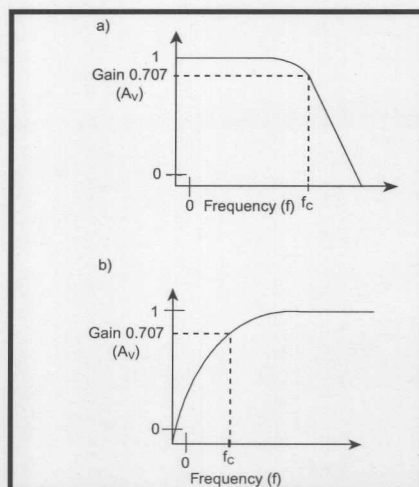
## IT'S JUST A PHASE

In addition to the amplitude of the output signal, you should also consider the phase shift between the input and output. The phase shift is a function of the real resistance and imaginary reactance in the filter. Because the reactance depends on the input frequency, the phase shift will change as the frequency changes. The passive low-pass filter has no phase shift at DC and reaches a maximum of -90° at frequencies well above the corner. The phase shift is -45° at the corner. Refer to Photo 1 to see the range of phase angles.

The high-pass filter has a phase shift of 90° at DC and decreases to 0° at frequencies well above the corner. Its phase shift is 45° at the corner.

## WHAT'S LOAD GOT TO DO WITH IT?

One disadvantage of the passive filter is the effect of a load resistor on its response. The gain of the low-pass filter and the high-pass filter decreases when a load resistor is connected



Figures 2a and b—Gain versus frequency for the passive low-pass and high-pass filters can be seen here. Note that each filter has a gain of 0.707 at the corner.

$$V_{OUT} = \left( \frac{X_C}{R + X_C} \right) V_{IN}$$

$$V_{OUT} = \left( \frac{1}{R + \frac{1}{j2\pi fC}} \right) V_{IN}$$

$$\frac{V_{OUT}}{V_{IN}} = A_V = \frac{1}{R + \frac{1}{j2\pi fC}} = \frac{1}{1 + j2\pi fRC}$$

$$\text{If } f_C = \frac{1}{2\pi RC}, \text{ then } A_V = \frac{1}{1 + j\left(\frac{f}{f_C}\right)}$$

$$\text{Magnitude of } A_V = \frac{1}{\sqrt{1 + \left(\frac{f}{f_C}\right)^2}}$$

**Figure 3**—Here is the derivation of the gain formula for the passive low-pass filter.  $j$  indicates that complex numbers are involved (and that the gain has real and imaginary components).

at the output. Even more interesting, the corner frequency also changes. In fact, it goes up. The example low-pass filter with a corner of 1591 Hz will see its corner increase to 1930 Hz when a 4.7-kilohm load resistor is connected.

To understand why this happens, examine the circuit transformations shown in Figure 4. The load resistor is in parallel with the filter capacitor. Because the order of the components in a parallel connection does not matter, you can easily swap the load resistor and filter capacitor. Then, using Thevenin analysis, reduce the two-resistor network to a single resistor, which will be equal to the parallel combination of the filter resistor and load resistor. It is this equivalent resistance that acts with the capacitor to determine the corner frequency. Because this equivalent resistance (824 ohms) is less than the original filter resistance (1000 ohms), the corner frequency of the loaded filter goes up.

In addition, the voltage-divider action of the filter resistor and the load resistor reduce the amount of signal that actually gets filtered. With a 4.7-kilohm load, the input signal is reduced by a factor of 0.824.

Recall the example filter with a 5- $V_{RMS}$ , 2-KHz input signal. The unloaded gain was 0.623 and the output voltage was 3.11  $V_{RMS}$ . When a 4.7-kilohm load resistor is added, the output voltage drops to 2.86  $V_{RMS}$ . This is because the signal is reduced

by 0.824 and the corner increases to 1930 Hz (giving a gain of 0.694).

The corner for the passive high-pass filter is affected in a similar way. There is no voltage-divider action, however, because the load resistor is placed directly in parallel with the filter resistor in the high-pass filter.

## ACTIVATE THAT FILTER!

The disadvantages of passive filters are overcome by active filters. Signals may pass through the active filter greatly amplified, and the filter may have a reasonable load attached without changing the corner frequency or the gain. Figures 5a-c show two ways to make an active low-pass filter as well as an active high-pass filter. All of these filters are first-order filters. They can be cascaded to make second-order or higher filters. Some special active filters (such as Sallen-Key) also use multiple feedback loops to obtain second-order response.

## FILTERS TO THE RESCUE

Several years ago, I designed an 8085-microprocessor timing system that was used to keep accurate time over a 24-h period. DIP switches on the circuit board selected the appropriate time intervals. I tapped the 12- $V_{RMS}$  power supply with a resistor and ran the signal through a transistor and two counters to get an accurate 1-Hz interrupt signal for the processor. I developed a prototype and mailed it to the client for testing. I was disturbed to get a call indicating that the system was running too fast, with a 24-h cycle only taking 16 h.

The problem turned out to be the wall clocks used in the testing lab. The clocks responded to 2- and 3-KHz tones placed onto the 60-Hz AC power line. This clock synchronization signal caused thousands of interrupts whenever they occurred, rather than the slow 1-Hz interrupts required for accurate timing.

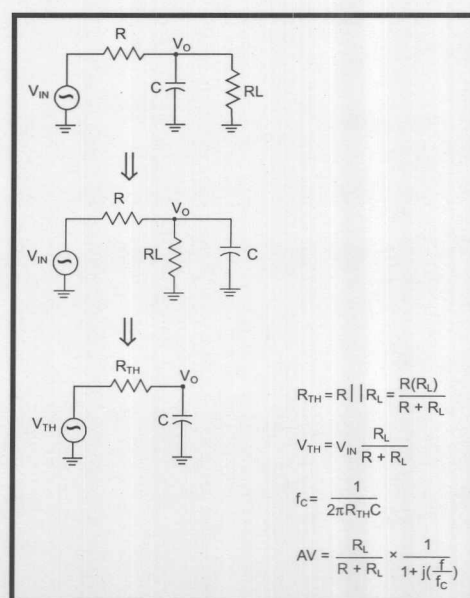
The fix was simple. I added a small capacitor to ground at the base of the transistor in the interrupt circuit. This formed a low-pass filter on the base of the transis-

tor, with the corner frequency set at 60 Hz. This allowed the low-frequency 60-Hz signal through and blocked the higher-frequency 2- and 3-KHz tones.

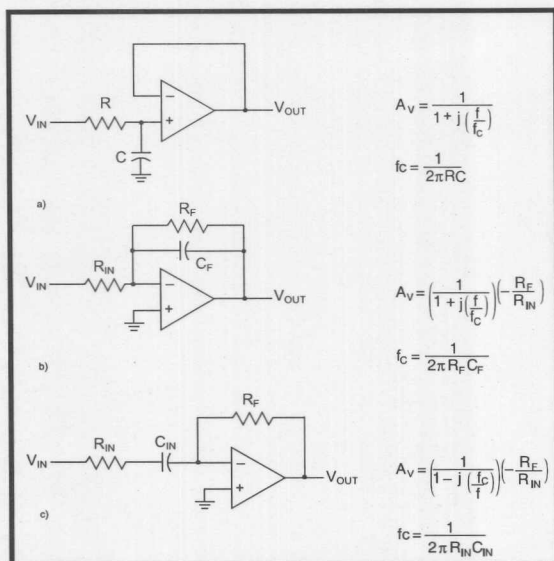
## LOOKING FURTHER

Round out your examination of filters by reviewing the many interesting filters available, such as Butterworth, Chebyshev, and Sallen-Key. Also, bear in mind that filtering is now also being performed digitally, on streams of numbers representing the input signal. For example, image processing systems use various filters to extract or enhance certain information in an image. So, filtering goes beyond the world of components and into the realm of software. Now, you've got a whole new world to learn about and enjoy. ■

*James Antonakos, a Professor in the Electrical Engineering Technology Department at Broome Community College, has over 25 years of experience designing digital and analog circuitry. He is also the author of numerous textbooks on microprocessors, programming, and microcomputer systems. You may reach him at [antonakos\\_j@sunybroome.edu](mailto:antonakos_j@sunybroome.edu) or visit his web site at [http://www.sunybroome.edu/~antonakos\\_j](http://www.sunybroome.edu/~antonakos_j).*



**Figure 4**—Rearranging and reducing the loaded low-pass filter to see how its gain is affected.



**Figures 5a-c**—Here you can see how an op-amp is used to make an active filter. Note that the inverting amplifier configuration adds its own 180° of phase shift to the output signal. The feedback and input resistors establish the maximum gain.

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